

Question # 1*Do yourself as Question # 2***Question # 2**

$$b = 12.5, \quad c = 23, \quad a = 38^\circ 20'$$

By law of cosine

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos a \\ &= (12.5)^2 + (23)^2 - 2(12.5)(23) \cos 38^\circ 20' \\ &= 156.25 + 529 - 575(0.7844) = 234.21 \end{aligned}$$

$$\Rightarrow a = \sqrt{234.21} \quad \Rightarrow \boxed{a = 15.304}$$

Again by law of cosine

$$\begin{aligned} \cos b &= \frac{c^2 + a^2 - b^2}{2ca} = \frac{(23)^2 + (15.304)^2 - (12.5)^2}{2(23)(15.304)} \\ &= \frac{529 + 234.21 - 156.25}{703.984} = \frac{606.96}{703.984} = 0.8622 \end{aligned}$$

$$b = \cos^{-1}(0.8622) \quad \Rightarrow \boxed{b = 30^\circ 26'}$$

Now

$$\begin{aligned} \cos g &= \frac{a^2 + b^2 - c^2}{2ab} = \frac{(15.304)^2 + (12.5)^2 - (23)^2}{2(15.304)(12.5)} \\ &= \frac{234.21 + 156.25 - 529}{382.6} = -\frac{138.538}{382.6} = -0.3621 \end{aligned}$$

$$b = \cos^{-1}(-0.3621) \quad \Rightarrow \boxed{b = 111^\circ 14'}$$

Question # 3

$$a = \sqrt{3} - 1 = 0.732, \quad b = \sqrt{3} + 1 = 2.732, \quad g = 60^\circ$$

By law of cosine

$$\begin{aligned} c^2 &= a^2 + b^2 - ab \cos g \\ &= (0.732)^2 + (2.732)^2 - 2(0.732)(2.732) \cos 60^\circ \\ &= 0.5358 + 7.4638 - 1.9998 = 5.9998 \approx 6 \end{aligned}$$

$$\Rightarrow \boxed{c = \sqrt{6} = 2.449}$$

Again by law of cosines

$$\begin{aligned} \cos a &= \frac{b^2 + c^2 - a^2}{2bc} = \frac{(2.732)^2 + (2.449)^2 - (0.732)^2}{2(2.732)(2.449)} \\ &= \frac{7.4638 + 5.9976 - 0.5358}{13.3813} = \frac{12.9256}{13.3813} = 0.9659 \end{aligned}$$

$$\Rightarrow a = \cos^{-1}(0.9659) \quad \Rightarrow \boxed{a = 15^\circ}$$

Since in any triangle

$$a + b + g = 180$$

$$\Rightarrow b = 180 - a - g = 180 - 15 - 60 \quad \Rightarrow \boxed{b = 105^\circ}$$

Question # 4 & 5*Do yourself as above*

Question # 6

$$a = 36.21, \quad b = 42.09, \quad g = 44^\circ 29'$$

Since $a + b + g = 180$

$$\Rightarrow a + b = 180 - g$$

$$= 180 - 44^\circ 29'$$

$$\Rightarrow a + b = 135^\circ 31' \dots\dots\dots (i)$$

By law of tangent

$$\frac{a - b}{a + b} = \frac{\tan\left(\frac{a - b}{2}\right)}{\tan\left(\frac{a + b}{2}\right)}$$

$$\Rightarrow \frac{36.21 - 42.09}{36.21 + 42.09} = \frac{\tan\left(\frac{a - b}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$$

$$\Rightarrow \frac{-5.88}{78.3} = \frac{\tan\left(\frac{a - b}{2}\right)}{\tan(67^\circ 45')} \quad \Rightarrow -0.0751 = \frac{\tan\left(\frac{a - b}{2}\right)}{2.4443}$$

$$\Rightarrow \tan\left(\frac{a - b}{2}\right) = -0.0751(2.4443)$$

$$= -0.1836$$

$$\Rightarrow \frac{a - b}{2} = \tan^{-1}(-0.1836)$$

$$\Rightarrow \frac{a - b}{2} = -10^\circ 24' \quad \Rightarrow a - b = -20^\circ 48' \dots\dots\dots (ii)$$

Adding (i) & (ii)

$$a + b = 135^\circ 31'$$

$$a - b = -20^\circ 48'$$

$$2a = 114^\circ 43' \quad \Rightarrow \boxed{a = 57^\circ 22'}$$

Putting value of a in eq. (i)

$$57^\circ 22' + b = 135^\circ 22'$$

$$\Rightarrow b = 135^\circ 22' - 57^\circ 22' \quad \Rightarrow \boxed{b = 78^\circ 9'}$$

Now by law of sine

$$\frac{c}{\sin g} = \frac{a}{\sin a}$$

$$\Rightarrow \frac{c}{\sin 44^\circ 29'} = \frac{36.21}{\sin 57^\circ 22'}$$

$$\Rightarrow c = \frac{36.21}{\sin 57^\circ 22'} \cdot \sin 44^\circ 29'$$

$$= \frac{36.21}{0.8421} \cdot 0.7007 \quad \Rightarrow \boxed{c = 30.13}$$

Question # 7, 8 & 9

Do yourself as above

Question # 10

$$\begin{aligned}
 b &= 61, \quad c = 32, \quad a = 59^\circ 30' \\
 \text{Since } a + b + g &= 180 \\
 \Rightarrow b + g &= 180 - a \\
 &= 180 - 59^\circ 30' \\
 \Rightarrow b + g &= 120^\circ 30' \dots\dots\dots (i)
 \end{aligned}$$

*Correction

By law of tangent

$$\begin{aligned}
 \frac{b-c}{b+c} &= \frac{\tan\left(\frac{b-g}{2}\right)}{\tan\left(\frac{b+g}{2}\right)} \Rightarrow \frac{61-32}{61+32} = \frac{\tan\left(\frac{b-g}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)} \\
 \Rightarrow \frac{29}{93} &= \frac{\tan\left(\frac{b-g}{2}\right)}{\tan(60^\circ 15')} \Rightarrow 0.3118 = \frac{\tan\left(\frac{b-g}{2}\right)}{1.7496} \\
 \Rightarrow \tan\left(\frac{b-g}{2}\right) &= 0.3118(1.7496) \\
 &= 0.5455 \\
 \Rightarrow \frac{b-g}{2} &= \tan^{-1}(0.5455) \\
 \Rightarrow \frac{b-g}{2} &= 28^\circ 37' \Rightarrow b-g = 57^\circ 14' \dots\dots\dots (ii)
 \end{aligned}$$

Adding (i) & (ii)

$$\begin{aligned}
 b + g &= 120^\circ 30' \\
 b - g &= 57^\circ 14' \\
 \hline
 2b &= 177^\circ 44' \Rightarrow \boxed{b = 88^\circ 52'}
 \end{aligned}$$

Putting value of a in eq. (i)

$$\begin{aligned}
 88^\circ 52' + g &= 120^\circ 30' \\
 \Rightarrow g &= 120^\circ 30' - 88^\circ 52' \Rightarrow \boxed{g = 31^\circ 38'}
 \end{aligned}$$

Now by law of sine

$$\begin{aligned}
 \frac{c}{\sin g} &= \frac{a}{\sin a} \\
 \Rightarrow \frac{c}{\sin 31^\circ 38'} &= \frac{36.21}{\sin 59^\circ 30'} \\
 \Rightarrow c &= \frac{36.21}{\sin 59^\circ 30'} \cdot \sin 31^\circ 38' \\
 &= \frac{36.21}{0.8616} \cdot 0.5244 \Rightarrow \boxed{c = 22.04}
 \end{aligned}$$

Question # 11Let $a : b = 3 : 2$

$$\text{i.e. } \frac{a}{b} = \frac{3}{2} \Rightarrow a = \frac{3}{2}b$$

and $g = 57^\circ$

$$\begin{aligned}
 \text{Since } a + b + g &= 180 \\
 \Rightarrow a + b &= 180 - g
 \end{aligned}$$

$$= 180 - 57 \Rightarrow a + b = 123^\circ \dots\dots\dots (i)$$

By law of tangent

$$\begin{aligned} \frac{a-b}{a+b} &= \frac{\tan\left(\frac{a-b}{2}\right)}{\tan\left(\frac{a+b}{2}\right)} \Rightarrow \frac{\frac{3}{2}b-b}{\frac{3}{2}b+b} = \frac{\tan\left(\frac{a-b}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)} \\ \Rightarrow \frac{\frac{1}{2}b}{\frac{5}{2}b} &= \frac{\tan\left(\frac{a-b}{2}\right)}{\tan(61^\circ 30')} \Rightarrow \frac{1}{5} = \frac{\tan\left(\frac{a-b}{2}\right)}{1.8418} \\ \Rightarrow \tan\left(\frac{a-b}{2}\right) &= \frac{1}{5}(1.8418) \\ &= 0.3684 \\ \Rightarrow \frac{a-b}{2} &= \tan^{-1}(0.3684) = 20^\circ 13' \\ \Rightarrow a-b &= 40^\circ 26' \dots\dots\dots (ii) \end{aligned}$$

Adding (i) & (ii)

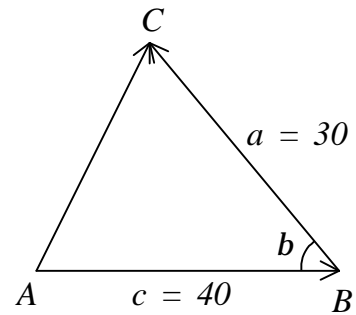
$$\begin{aligned} a + b &= 123^\circ \\ a - b &= 40^\circ 27' \\ \hline 2a &= 163^\circ 27' \Rightarrow \boxed{a = 81^\circ 44'} \end{aligned}$$

Putting value of a in eq. (i)

$$\begin{aligned} 81^\circ 44' + b &= 123^\circ \\ \Rightarrow b &= 123^\circ - 81^\circ 44' \Rightarrow \boxed{b = 41^\circ 16'} \end{aligned}$$

Question # 12

Since $\overline{AB} = c = 40N$
 $\overline{BC} = a = 30N$
 $m\angle B = b = 147^\circ 25'$
 $\overline{AC} = b = ?$



By law of cosine

$$\begin{aligned} b^2 &= c^2 + a^2 - 2ca \cos b \\ &= (40)^2 + (30)^2 - 2(40)(30)\cos 147^\circ 25' \\ &= 1600 + 900 - 2400(-0.8426) \\ &= 4522.26 \\ \Rightarrow b &= \sqrt{4522.26} = 67.248 \end{aligned}$$

i.e. $\overline{AC} = 67.248N$

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