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Mathematics B-Course 2008
UNIVERSITY OF THE PUNJAB, LAHORE
Available online @ http://www.MathCity.org/papers, Paper A

Subject: Mathematics-B Course **TIME ALLOWED: 3 hrs.**
PAPER- A **MAX. MARKS: 100**

Attempt SIX questions by selecting ONE question from Section-I, TWO questions from Section-II, TWO questions from Section-III and One question from Section-IV.

SECTION - I

Q.1. (a) Prove that $\frac{\bar{a}}{a}+\frac{\bar{b}}{b}$ is equally inclined with $\bar{a}$ and $\bar{b}$. 4+6+6

(b) Solve the following equation $a_r x + b_r y + c_r z = d_r$ $r = 1, 2, 3$

(c) If $\bar{a}, \bar{b}, \bar{c}$ and $\bar{a}', \bar{b}', \bar{c}'$ are reciprocal system of vectors, then show that

$$\bar{a}' \times \bar{b}' + \bar{b}' \times \bar{c}' + \bar{c}' \times \bar{a}' = \frac{\bar{a}+\bar{b}+\bar{c}}{(\bar{a}\ \bar{b}\ \bar{c})}$$

Q.2. (a) The necessary and sufficient condition for the vector $\bar{a}$ to have a constant magnitude is $\bar{a} \cdot \frac{d\bar{a}}{dt} = 0$ 4+6+6

(b) Solve $\bar{a} \times \frac{d^2\bar{v}}{dt^2} = \bar{b}$ where $\bar{a}, \bar{b}$ are constant vectors and $\bar{v}$ is a vector function of t.

(c) Show that curl grad $\phi = 0$

SECTION - II

Q.3. (a) A single force and a couple acting in the same plane upon a rigid body are equivalent to a single force acting in a direction parallel to its original direction. 8+9

(b) Forces $2\overline{BC}, \overline{CA}, \overline{BA}$ act along the sides of ΔABC. Show that their resultant is $6\overline{DE}$ where D bisects $\overline{BC}$ and E is a point on $\overline{CA}$ such that

$$CE = \frac{1}{3}CA$$

Q.4. (a) If forces $\ell\overline{AB}, m\overline{BC}, \ell\overline{CD}, m\overline{DA}$ acting along the sides of a quadrilateral are equivalent to a couple. Show that either $\ell = m$ or ABCD is a // gram. 8+9

(b) A uniform rod of length 2a rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall, show that in the position of Equilibrium the beam is inclined to wall at an angle $\left(\text{Sin}^{-1}\frac{b}{a}\right)^{1/3}$.

Q.5. (a) One end of a uniform ladder of weight w rests against a smooth wall and other end on a rough ground which slopes down from the wall at angle α to the horizontal. Find the inclination of the ladder to the horizontal when it is at the point of sliding and show that the reaction of the wall is then w tan $(\lambda - \alpha)$ where λ is the angle of friction. 8+9

(b) A uniform rod slides with its end on two fixed equally rough rods are being vertical and other inclined at an angle α to the horizonal. Show that the angle θ to horizontal of movable rod when it is on the point of sliding is given by $\tan\theta = \dfrac{1-2\mu\tan\alpha-\mu^2}{2(\mu+\tan\alpha)}$

Q.6. (a) Find C·G of lamina in the term of a vector of a circle subtending an angle 2α at the centre. 8+9

(b) The radius of the faces of a frustum of a solid cone are 2 feet and 3 feet and the height of the frustum is 4 feet. Find the distance of the C·G from the larger face.

SECTION - III

Q.7. (a) A particle is moving with uniform speed 'V' along the curve 8+9

$$x^2y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$$

Show that its acceleration has the Maximum Value $\dfrac{10V^2}{9a}$

(b) Show that the total work done on a particle in moving it along a curve C from P_1 to P_2 is equal to the increase $T_2 - T_1$ in the K·E at time t_1 and t_2 corresponding to the positions P_1 and P_2.

Q.8. (a) Determine whether $\bar{F} = (x^2y - z^3)\bar{i} + (3xyz + xz^2)\bar{j} + (2x^2yz + yz^4)\bar{k}$ is conservative. 8+9

(b) If a point P moves with a Velocity 'V' given by $v^2 = n^2(ax^2 + 2bx + c)$ show that P executes SHM. Find the centre, the amplitude and the time period of Motion.

Q.9. (a) Prove that the path of Projectile is a parabola. 8+9

(b) A shell bursts on contact with the ground and pieces from it fly in all direction with all speeds up to 80 feet per second. Prove that a man 100 feet way is in danger for $\dfrac{5}{\sqrt{2}}$ seconds.

Q.10. (a) Find the diffrential Equation of the orbit of a particle in Pedal 8+9

form i.e. $\dfrac{h^2}{P^3}\dfrac{dP}{dr} = F$

(b) The law of force in Mu^5 and a particle is projected from an apse at a distance 'a'. Find the orbit when the velocity of Projection is $\dfrac{\sqrt{M}}{a^2}$

SECTION - IV

Q.11. (a) Define Impulse from Newton's 2nd law. 8+8

(b) If two equal and perfectly elastic spheres moving with velocity u and u' impinge directly. Show that the velocities interchange after impact.

Q.12. (a) At what angle must a body whose elasticity is $\dfrac{1}{3}$ be inclined on perfectly hard plane so that the angle between the direction before and after impact may be right angle. 8+8

(b) A particle of elasticity e is projected in a direction inclined to the Vertical and bounces on smooth horizontal plane. The range of 1st rebound is 'r'. Find the range of next.

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